## MA114 Summer 2018

## Worksheet 13 - Power Series Part 1 - 7/03/18

1. (a) What do the terms power series, radius of convergence, and interval of convergence mean?
(b) Find a formula for the coefficients $c_{k}$ of the power series

$$
\frac{1}{0!}+\frac{2}{1!} x+\frac{3}{2!} x^{2}+\frac{4}{3!} x^{3}+\ldots
$$

(c) Find a formula for the coefficients $c_{n}$ of the power series

$$
1+2 x+3 x^{2}+4 x^{3}+5 x^{4}+\ldots .
$$

(d) For what values of $x$ does the series $\sum_{n=1}^{\infty} 2(\cos (x))^{n-1}$ converge?
(e) Suppose $\lim _{n \rightarrow \infty} \sqrt[n]{\left|c_{n}\right|}=c$ where $c \neq 0$. What is the radius of convergence of the power series $\sum_{n=0}^{\infty} c_{n} x^{n}$ ?
(f) Consider the function $\frac{5}{1-x}$. Find a power series that is equal to $f(x)$ for every $x$ such that $|x|<1$.
2. Find the radius and interval of convergence for each power series. Remember to check the endpoints.
(a) $\sum_{n=0}^{\infty}(5 x)^{n}$
(b) $\sum_{n=0}^{\infty} \frac{(-1)^{n} n}{4^{n}}(x-3)^{n}$
(c) $\sum_{n=0}^{\infty} \frac{x^{2 n}}{(-3)^{n}}$
(d) $\sum_{n=0}^{\infty} \frac{(x-2)^{n}}{n^{n}}$
(e) $\sum_{n=0}^{\infty} n!(x-1)^{n}$

